

Bulk Kalb-Ramond field in Randall Sundrum scenario

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Abstract

We have considered the most general gauge invariant five-dimensional action of a second rank antisymmetric Kalb-Ramond tensor gauge theory, including a topological term of the form $\epsilon^{ABLMN} B_{AB} H_{LMN}$ in a Randall-Sundrum scenario. Such a tensor field B_{AB} (whose rank-3 field strength tensor is H_{LMN}), which appears in the massless sector of a heterotic string theory, is assumed to coexist with the gravity in the bulk. The third rank field strength corresponding to the Kalb-Ramond field has a well-known geometric interpretation as the spacetime torsion. The only non-trivial classical solutions corresponding to the effective four-dimensional action are found to be self-dual or anti-selfdual Kalb-Ramond fields. This ensures that the four-dimensional effective action on the brane is parity-conserving. The massive modes for both cases, lying in the TeV range, are related to the fundamental parameters of the theory. These modes can be within the kinematic reach of forthcoming TeV scale experiments. However,

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the couplings of the massless as well as massive Kalb-Ramond modes with matter on the visible brane are found to be suppressed vis-a-vis that of the graviton by the warp factor, whence the conclusion is that both the massless and the massive torsion modes appear much weaker than curvature to an observer on the visible brane.

An interesting idea floated in recent times is that our observable universe lies on a (3+1)-dimensional brane embedded in a higher dimensional ('bulk') spacetime. Such theories, involving compact spacelike extra dimensions, have shot into prominence by providing a natural explanation of the Planck-weak hierarchy problem. The possible presence of TeV scale effects in such models have made this study worthwhile.

Theories of this type fall into two broad categories. One category includes the kind of models proposed by Arkani-Hamed, Dimopoulos and Dvali[1] where a factorisable geometry of the compact dimensions is postulated, and the Planck scale gets related to the TeV-scale 'bulk' gravitational coupling via the volume of the internal dimensions. In the other category we have the proposal by Randall and Sundrum[2]. Here the hierarchy is generated by an exponential function, called the warp factor, introduced in the metric in a non-factorisable geometry. In minimal versions of both the scenarios, all matter fields reside on the branes while gravity propagates in the bulk. The observable effects are generated by the Kaluza-Klein projection of the bulk gravity on the brane, where the spacings of the modes and the interactions with the standard model (SM) fields are governed by the dynamics of the respective models.

The present work concerns the Randall-Sundrum scenario. Here, the extra dimension is a Z_2 orbifold of radius r_c . There are two branes at the two orbifold fixed points, viz. $\phi = 0$ and $\phi = \pi$, where the compact co-ordinate x_4 is given by $x_4 = \phi r_c$. All visible matter resides on the brane at $\phi = \pi$ (called the 'visible brane' or 'TeV brane'), while gravity peaks on the brane at $\phi = 0$ (called the 'Planck brane' or the 'hidden brane'). The five-dimensional metric in this scenario can be written as

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \quad -\pi \leq \phi \leq \pi \quad (1)$$

with $\eta_{\mu\nu} = (-, +, +, +)$, $\sigma = kr_c|\phi|$ and k is of the order of the Planck mass M_P . The Planck scale in four dimension (M_P) and higher dimension (M) is related through

$$M_P^2 = \frac{M^3}{k} [1 - e^{-2kr_c\pi}] \quad (2)$$

M_P , M and k are all of the same order of magnitude. In this warped spacetime five-dimensional mass scales are related to the effective four-dimensional masses through the 'warp factor' $e^{-kr_c|\phi|}$. For $kr_c \simeq 12$, TeV scale mass parameters of the form $m = Me^{-kr_c\pi}$ thus arise from the Planck scale on the 'standard model' brane ($\phi = \pi$).

Implications of other types of bulk fields, such as scalars, gauge fields and fermions, [3]-[7] have already been explored as an extension of the above theory. We focus in this paper on the case where bulk spacetime is attributed with torsion, in the same way as in

Einstein-Cartan theories of four-dimensional gravity. The consequences of such bulk torsions have earlier been studied from different angles [8, 9, 10, 11]. In the last of these references, we studied the consequence of relating torsion to the rank-2 antisymmetric Kalb-Ramond field that occurs as a massless excitation in string theories. Here we carry that study to a conclusion, using the most general five-dimensional action as our starting point, and coming up with some novel observations regarding the KR field itself in such scenarios.

As torsion is an inescapable consequence of matter field with spin, studies preoccupied with extra dimensions have sometimes related it [8, 9] to fermion fields residing either on the brane or in the bulk. In a different approach[10, 11] motivated by string theories [12], torsion is considered to have the same status as gravity in the bulk. Such consideration can be justified by remembering that torsion is commonly incorporated into the theory by adding an antisymmetric part in the affine connection and can couple to all matter fields with spin. This is the essence of Einstein Cartan type theories[13]. In our approach, the source of torsion is traced to the rank-2 antisymmetric Kalb Ramond (KR) field B_{MN} , which arises as a massless mode in heterotic string theories. Thus we require the graviton and the KR field to coexist in the bulk. In a sense, this amounts to consigning to the bulk both the characteristics of spacetime, namely, curvature and torsion.

To be precise, the rank-3 antisymmetric tensor required for introducing torsion is found in H_{MNL} which is taken to be the strength of the KR field B_{MN} [14]:

$$H_{MNL} = \partial_{[M} B_{NL]} \quad (3)$$

where both H_{MNL} and B_{MN} lie in the bulk while matter fields reside on brane. In this approach, we have analysed earlier both the ADD[10] and RS[11] scenarios. A striking consequence of considering bulk torsion in RS scenario is that the torsion zero mode is enormously suppressed through the warp factor on the visible brane with respect to gravity and thus could provide the illusion of the torsionless universe.

In our earlier analysis of the RS scenario, we have considered only the minimal way of including torsion in the 5 dimensional action:

$$\mathcal{S}_G = \int d^4x \int d\phi \sqrt{-G} \, 2[M^3 R(G) - H_{MNL} H^{MNL}] \quad (4)$$

In this work we would like to extend the 5 dimensional action with a term of the form $\epsilon^{ABMNL} B_{AB} H_{MNL}$ so that we get the most general 5 dimensional action which is invariant under the Kalb-Ramond gauge transformation, namely, $(\delta B_{MN} = \partial_{[M} \lambda_{N]})$ [15]:

$$\mathcal{S}_G = \int d^4x \int d\phi \sqrt{-G} \, 2[M^3 R(G) - H_{MNL} H^{MNL} - 2M_0 \epsilon^{ABMNL} B_{AB} H_{MNL}] \quad (5)$$

Here M_0 is a coupling parameter with the dimension of mass (\sim five dimensional Planck mass M). At this point it is worth mentioning that such a term is purely topological in five dimensional KR gauge sector and is analogous to the Chern-Simons term $\epsilon^{ijk} A_i F_{jk}$ in the $U(1)$ gauge sector in three dimension. We thus explore in this work the effect of such a topological term on the effective four dimensional compactified theory in a Randall-Sundrum compactification scenario. One also finds in the literature similar-looking terms constructed out of a $U(1)$ gauge field in N=2, D=5 Supergravity theories [16]. Here we focus on such a term originating in the Kalb-Ramond gauge sector. Another interesting point to note here is that a term of the above appearance in N=2, D=5 Supergravity theories would break parity in even space-time dimensions, and thus it may be curious to check whether the inclusion of KR topological term also leads to violation of parity when dimensional compactification is performed. If it does, then it would lead to a fresh source of parity violation in the domain of gravity. It has been shown that such a source of parity violation may lead to phenomena like fermion helicity flip and Cosmic Microwave background anisotropy[17].

We start with the KR part of the above five-dimensional action which, upto a dimensionless multiplicative factor, can be written as

$$\mathcal{S}_H = \int d^4x \int d\phi \sqrt{-G} [H_{MNL} H^{MNL} + 2M_0 \epsilon^{ABMNL} B_{AB} H_{MNL}] \quad (6)$$

where H_{MNL} is related to B_{MN} by equation (3). We follow the convention where Latin indices run from 0 to 4 whereas Greek indices run from 0 to 3. Furthermore we use the KR gauge freedom to set $B_{4\mu} = 0$. This allows us to get rid of massive vector modes on the brane. Thus we are left with the KR components corresponding to brane indices only, which, however, are functions of both compact and noncompact coordinates.

Using the metric given in (1) we write the above action in the following form

$$\begin{aligned} \mathcal{S}_H = \int d^4x \int d\phi r_c e^{2\sigma(\phi)} \quad & \left[\eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H_{\mu\nu\lambda} H_{\alpha\beta\gamma} - \frac{3}{r_c^2} e^{-2\sigma(\phi)} \eta^{\mu\alpha} \eta^{\nu\beta} B_{\mu\nu} \partial_\phi^2 B_{\alpha\beta} \right. \\ & \left. + \frac{6M_0}{r_c} e^{-2\sigma(\phi)} \mathcal{E}^{5\mu\nu\alpha\beta} B_{\alpha\beta} \partial_\phi B_{\mu\nu} \right] \end{aligned} \quad (7)$$

Here we have used the relation

$$\epsilon^{MNLAB} = \frac{\mathcal{E}^{MNLAB}}{\sqrt{-G}} \quad (8)$$

where \mathcal{E}^{MNLAB} is the Levi-Civita tensor density in 5 dimension while ϵ^{MNLAB} is the tensor formed from the Levi-Civita tensor density[18].

Assuming the following decomposition for the Kalb-Ramond field[11]:

$$B_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} B_{\mu\nu}^n(x) \frac{\chi^n(\phi)}{\sqrt{r_c}} \quad (9)$$

one can recast the five dimensional action in (7) as

$$\begin{aligned} \mathcal{S}_H = \int d^4x \sum_{n=0}^{\infty} [& \eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\lambda\gamma}H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + 3\eta^{\mu\alpha}\eta^{\nu\beta}B_{\mu\nu}^n B_{\alpha\beta}^n (-\frac{e^{-2\sigma}}{r_c^2} \frac{1}{\chi^n} \partial_\phi \partial_\phi \chi^n) \\ & + 6\epsilon^{\mu\nu\alpha\beta}B_{\mu\nu}^n B_{\alpha\beta}^n (\frac{e^{-2\sigma}}{r_c} \frac{M_0}{\chi^n} \partial_\phi \chi^n)] \end{aligned} \quad (10)$$

provided that the modes χ^n satisfy the orthonormality condition

$$\int_{-\pi}^{\pi} e^{2\sigma(\phi)} \chi^m(\phi) \chi^n(\phi) d\phi = \delta_{mn} \quad (11)$$

In equation (10) with a 4D Minkowski metric, the Levi-Civita tensor density is replaced by the 4 dimensional Levi-Civita tensor with the help of the relation mentioned in (8). It is interesting to notice that the effective action in four-dimensions contains, apart from the kinetic term and the mass term ($B_{\mu\nu}^n B^{\mu\nu}$) for the Kalb-Ramond field, an additional term of the form $B_{\mu\nu}^n \tilde{B}^{\mu\nu}$ where $\tilde{B}^{\mu\nu} (= \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}^n)$ is the dual of Kalb-Ramond field.

On solving the equation of motion from this effective four dimensional action (10), it is quite straightforward to find the solution for the Kalb-Ramond field. We found that the only non-trivial solution corresponds to self-dual or anti-dual Kalb-Ramond fields i.e, $B_{\mu\nu}^n = \tilde{B}_n^{\mu\nu}$ or $B_{\mu\nu}^n = -\tilde{B}_n^{\mu\nu}$. We have presented the proof of this result in the appendix. Such self-dual or anti self-dual conditions imply the reduction in the degrees of freedom of the KR field. Such a reduction originates in the additional five dimensional topological term and will have crucial consequences, as will be demonstrated in what follows.

The above conclusion tells us that, in terms of the classical solutions for the KR tower $B_{\mu\nu}^n$, the effective four dimensional action (10) can be finally expressed as

$$\mathcal{S}_H = \int d^4x \sum_{n=0}^{\infty} [\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\lambda\gamma}H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + 3m_{n\pm}^2 \eta^{\mu\alpha}\eta^{\nu\beta}B_{\mu\nu}^n B_{\alpha\beta}^n] \quad (12)$$

where $m_{n\pm}^2$ satisfies the following differential equations,

$$-\frac{1}{r_c^2} \frac{d^2 \chi^n}{d\phi^2} + \frac{2M_0}{r_c} \frac{d\chi^n}{d\phi} = m_{n+}^2 \chi^n e^{2\sigma} \quad (13)$$

for the dual Kalb-Ramond field ($B_{\mu\nu}^n = \tilde{B}_n^{\mu\nu}$) and

$$-\frac{1}{r_c^2} \frac{d^2 \chi^n}{d\phi^2} - \frac{2M_0}{r_c} \frac{d\chi^n}{d\phi} = m_{n-}^2 \chi^n e^{2\sigma} \quad (14)$$

for the antidual Kalb-Ramondfield ($B_{\mu\nu}^n = -\tilde{B}_n^{\mu\nu}$). Evidently, $\sqrt{3}m_{n+}$ and $\sqrt{3}m_{n-}$ gives the mass of the nth mode of the self-dual field and anti-dual field respectively. An interesting point to be noted here is that in the effective four-dimensional action (12) parity has

been finally found to be conserved though we started with a term involving the completely antisymmetric tensor in higher dimensions. We shall comment further on this at the end.

Next, we rewrite the above equations in terms of another parameter $z_n = \frac{m_n}{k} e^{\sigma(\phi)}$. For the dual Kalb-Ramond field with masses m_{n+} equation (13) takes a form which resembles the transformed Bessel differential equation

$$\left[z_n^2 \frac{d^2}{dz_n^2} + \left(1 - \frac{2M_0}{k}\right) z_n \frac{d}{dz_n} + z_n^2 \right] \chi^n = 0 \quad (15)$$

admitting a solution of Bessel and Neuman function of order $\nu = \frac{2M_0}{k}$

$$\chi^n = \frac{z_n^\nu}{N_n} [J_\nu(z_n) + \alpha_n Y_\nu(z_n)] \quad (16)$$

where N_n is the normalisation constant and α_n are the constant coefficients to be determined from the boundary conditions. Likewise, in the anti dual case, equation (14) can also be converted into a transformed Bessel differential equation

$$\left[z_n^2 \frac{d^2}{dz_n^2} + \left(1 + \frac{2M_0}{k}\right) z_n \frac{d}{dz_n} + z_n^2 \right] \chi^n = 0 \quad (17)$$

with a similar kind of solution

$$\chi^n = \frac{z_n^{-\nu}}{N_n} [J_\nu(z_n) + \alpha_n Y_\nu(z_n)] \quad (18)$$

of order $\nu = \frac{2M_0}{k}$. N_n is the overall normalisation, over and above the constant α_n . As mentioned earlier, we determine these constants from the boundary conditions which requires the first derivative of χ^n to be continuous at the orbifold fixed points $\phi = 0$ and $\phi = \pm\pi$ so that the self-adjointness of the left hand side of equation (13) (for the dual Kalb-Ramond field or (14) in case of antidual field) holds. From the continuity condition at $\phi = 0$ along with the limit $e^{kr_c\pi} \gg 1$ and the requirement that the mass values m_n on the brane should be of the order of TeV ($\ll k$), we obtain

$$\alpha_n \simeq \left[\frac{1}{\sqrt{\nu - 2}(\nu - 1)!} \left(\frac{x_n}{2} e^{-2kr_c\pi} \right)^{\nu-1} \right]^2 \quad (19)$$

for the self dual field with $x_n = z_n(\pi)$. Similarly, α_n for the antidual field is given by

$$\alpha_n \simeq \left[\frac{1}{\nu!} \left(\frac{x_n}{2} e^{-2kr_c\pi} \right)^{\nu+1} \right]^2 \quad (20)$$

It is quite apparent that for both the cases $\alpha_n \ll 1$ in the limit mentioned above. Therefore, in this approximation, the solution for χ^n presented in equations (16) and (18) for the two cases now becomes

$$\chi^n = \frac{z_n^\nu}{N_n} J_\nu(z_n) \quad \text{and} \quad \chi^n = \frac{z_n^{-\nu}}{N_n} J_\nu(z_n) \quad (21)$$

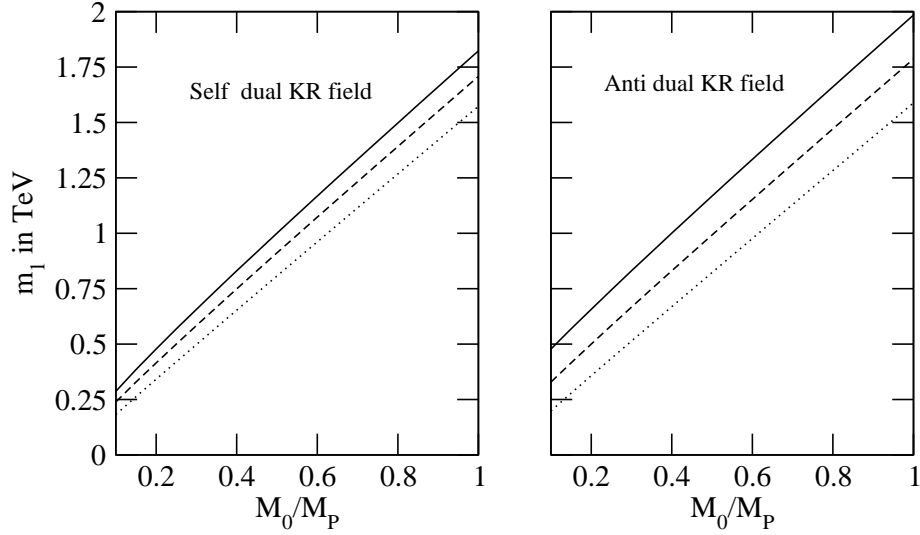


Figure 1: Plot of the masses m_1 in TeV scale with respect to M_0/M_P where $M_P = 10^{19}$ and $kr_c = 12$ and with different ratios for M_P/k (such as $M_P/k = 10, 20$ and 100 from the upper to the lower curves respectively in both figures). The first figure represents the mass of the self dual field for the lowest root of the Bessel function $J_{\nu-1}(x) = 0$ where $\nu = 2M_0/k$, while the second one gives the masses of the antidual field, where the order of the Bessel function is $2M_0/k + 1$.

respectively, where N_n are the normalisation constants in the two cases determined by the orthonormality condition (11). Next, we consider the continuity condition at $\phi = \pi$, which yields

$$J_{\nu-1}(x_n) = 0 \quad \text{and} \quad J_{\nu+1}(x_n) = 0 \quad (22)$$

for the dual and anti-dual field respectively. So, x_n is simply the root of the corresponding Bessel function. Consequently the mass spectrum in both the cases depends on the corresponding Bessel function. Thus, with values of x_n of the order of unity, the mass eigenvalues of the higher KK excitation $\sqrt{3}x_n k e^{-kr_c\pi}$ settle at the TeV range, as expected. One important point to note here is that the value of x_n are function of ν i.e, the ratio of the coupling mass parameter M_0 to k . As mentioned in the beginning both M_0 and k are on the order of the Plank scale M_P , but fractional difference also changes the eigenvalues of the mass spectrum. We present in figure 1 lowest possible massive mode as a function of the ratio M_0/M_P , with three values of the ratio M_P/k in each case. The lower limit, *i.e.* ($M_P/k = 10$), arises from the constraint that the scalar curvature in 5-dimensions exceeds M_P^2 for higher values of k .

Using the expression for χ^n in (11) one can find the normalisation constant. For the

selfdual case

$$N_n = \left[\int_{-\pi}^{\pi} d\phi e^{2\sigma(\phi)} z_n^{2\nu} J_\nu(z_n)^2 \right]^{\frac{1}{2}} \quad (23)$$

and for the anti selfdual field the normalisation condition yields

$$N_n = \left[\int_{-\pi}^{\pi} d\phi e^{2\sigma(\phi)} z_n^{-2\nu} J_\nu(z_n)^2 \right]^{\frac{1}{2}} \quad (24)$$

So far we were confined with the massive modes of the KR field and found that with the inclusion of the Chern Simons like term the masses of higher KK modes depends on the coupling parameter M_0 . Now, let us find the solution for the massless mode of the four dimensional projection of the KR field for both cases. Solving equation (13) and (14) for the massless KK mode, the zero mode of χ^n turn out to be

$$\chi^0 = c_1 + \frac{c_2}{2M_0 r_c} e^{2M_0 r_c |\phi|} \quad \text{and} \quad \chi^0 = c_1 - \frac{c_2}{2M_0 r_c} e^{-2M_0 r_c |\phi|} \quad (25)$$

for the self dual and anti dual field respectively. The imposition of self-adjointness forces us to a constant solution for χ^0 . Using the orthonormality condition, one thus obtains,

$$\chi^0 = \sqrt{kr_c} e^{-kr_c \pi} \quad (26)$$

for both the cases. Thus the zero mode exhibits a suppression by the warp factor. This result is identical to the one found in [11]. Thus with the most general Kalb Ramond gauge invariant action also we still conclude that the massless KR mode is enormously suppressed on the visible brane, ensuring the absence of torsion.

At this point it is quite natural to make a comparison with the case where the topological term is absent. As has been mentioned earlier, the masses now depend crucially on M_0/k as compared to the case presented in [11] where a minimal torsion term was present in the 5D action along with gravity and the masses of the higher KK modes were given by the zeros of first order Bessel function $J_1(x_n)$. The inclusion of the topological term in the action has shifted the mass spectrum from the earlier case but it is still very much in the TeV range and within the reach of the TeV-scale collider experiments. In [11], it was claimed that the masses of the KR tower are just given by a factor of $\sqrt{3}$ scaling from the graviton modes[19], whereas in this case the corresponding masses also depend upon the parameter M_0 . For a better comparison, in figure 2 we present some plots for the massive spectrum m_n against M_0/k in the three cases (with the minimal torsion term[11], self-dual and anti-dual KR field). The dashed lines correspond to the earlier case with the minimal torsion term ($J_1(x_n)$), the bold lines, to the self-dual KR field ($J_{\nu-1}(x_n)$), and the dotted lines, to the anti-dual field ($J_{\nu+1}(x_n)$, respectively). The three different plots correspond to the three lowest roots of the corresponding Bessel function. As is evident from the plot, both in the dual and the antidual

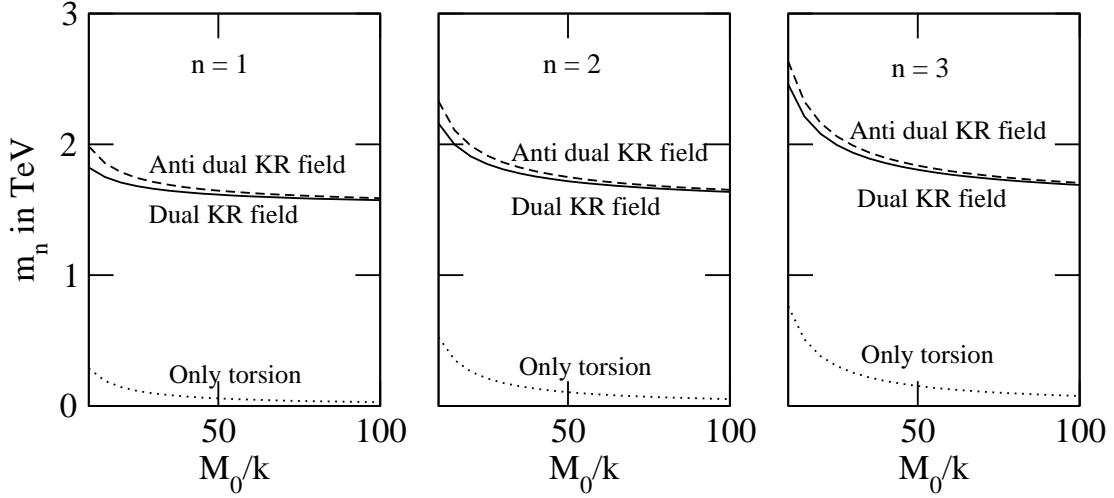


Figure 2: Plot of the three lowest mass eigenvalues of the higher KK modes with respect to M_0/k for the three lowest root of the Bessel function. The lowest line (dotted line) correspond to the case with torsion only[11] with $J_1(x) = 0$. The second line from the bottom (bold line) correspond to the case with dual KR field (*i.e.*, $J_{\nu-1}(x) = 0$ where $\nu = 2M_0/k$) and the uppermost line (dashed line) correspond to the antidual KR field (*i.e.*, $J_{\nu+1}(x) = 0$). Here also we have chosen $M_0 = M_P = 10^{19}$, with $kr_c = 12$.

cases the mass eigenvalues are boosted with the respect to the cases in [11], although they are still within the kinematic reach of TeV-scale experiments. However, their ‘visibility’ in the scenario considered here is drastically reduced, as discussed below.

As has already been indicated, the couplings of the massless modes with matter modes in this case are of the same kind as in reference [11]. For massive fields, the difference caused by the introduction of the ϵBH term becomes obvious if we consider the interactions with fermion fields, and compare them with equation (26) of [11].

For self-dual KR field, the coupling looks as follows:

$$\mathcal{L}_{\psi\bar{\psi}H} = -\bar{\psi}[i\gamma^\mu\sigma^{\nu\lambda}\left\{\frac{1}{M_P e^{kr_c\pi}}H_{\mu\nu\lambda}^0 + \frac{1}{\sqrt{12}M_P}\sum_{n=1}^{\infty}\frac{x_n^\nu}{N_n}J_\nu(x_n)H_{\mu\nu\lambda}^n\right\}]\psi \quad (27)$$

while for an antidual field, the expression is

$$\mathcal{L}_{\psi\bar{\psi}H} = -\bar{\psi}[i\gamma^\mu\sigma^{\nu\lambda}\left\{\frac{1}{M_P e^{kr_c\pi}}H_{\mu\nu\lambda}^0 + \frac{1}{\sqrt{12}M_P}\sum_{n=1}^{\infty}\frac{x_n^{-\nu}}{N_n}J_\nu(x_n)H_{\mu\nu\lambda}^n\right\}]\psi \quad (28)$$

using $kr_c = 12$.

The remarkable point to note here is that the massive mode coupling to fermion, as

given by equations (27) and (28), are drastically reduced compared to the corresponding case *without the presence of the ϵBH term*. Basically, the different co-efficients arising from the solutions of equations (13) and (14), are responsible for this. This feature has its origin in the second term on the left-hand side of each of them, which in turn owes itself to the last term in the action given in equation (5). It appears as if the large coefficient M_0 in the additional five dimensional topological term ϵBH causes the KR modes to decouple from all visible physics on the brane, although a tower within the kinematic reach of accelerator experiments is still around.

The analysis presented can be summarised as follows: We have considered the most general five-dimensional action invariant under Kalb Ramond gauge transformation where gravity and torsion are accorded the same status in the bulk. This is a kind of the generalisation of the action presented in [11]. The compactification of the fifth dimension gives rise to a spectrum of Kaluza Klein modes for the KR field. On solving the equation of motion in four dimensions, we find that the only non-trivial classical solutions correspond to self-dual or anti-dual KR field. This result turns out to be crucial for the effective four dimensional theory in the following ways:

(a) In spite of the inclusion of a five dimensional Levi-Civita term in the action, the effective four dimensional action after Randall-Sundrum compactification continues to be parity invariant. One expects that a parity violating interaction will be generated from the 4-dimensional Levi-Civita dependent term which appears from the five-dimensional Levi-Civita term after the RS compactification. However the KR field equations constrains the KR field to be selfdual or anti-selfdual only and thereby reduces the number of degrees of freedom and thus removes the compactified four dimensional Levi-Civita from the effective 4d action. As a result the effective 4d theory continues to be parity invariant. This is in perfect agreement with our earlier findings that parity seems to be protected by duality in all such scenarios[20, 21, 10]. (b) The massive KK modes spectrum for both cases depends on the coupling parameter M_0 and are shifted from the earlier case as expected, due to the inclusion of the additional term. But the mass eigenvalues of the spectrum are still on the order of TeV. However, the massive modes now have drastically reduced interaction strength with matter fields, and this practically destroys the scope for their detection in accelerator experiments. Such a reduction of coupling of the massive modes stems from selfdual or anti-selfdual nature of the KR field which has originate from the additional topological term with the large coefficient M_0 in the action. In other words, whereas a minimal theory [11] predicts an invisible zero-mode torsion but envisions potentially observable massive modes, the existence of the ϵBH -term makes all modes, massless or massive, practically invisible. However, the most significant result obtained in [11], namely, the massless mode of the four

dimensional projection of the Kalb Ramond field has an added suppression through the warp factor with respect to gravity in its interaction with other matter field remains valid in this modified scenario also. In addition the present scenario restricts the KR field to be selfdual or anti-selfdual. Therefore, starting from this more generalised action in five dimensions we still have a parity-conserving effective torsionless universe when we consider the projection of massless torsion mode on the visible brane.

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Appendix:

In the appendix we derive the solution of the KR field on solving the equation of motion in 4D. After integrating out the extra dimension we are left with the action (10) in the effective four dimension. The Lagrangian formed from action (10) is the following,

$$\mathcal{L}_B = \partial_{[\mu} B_{\nu\lambda]}^n \partial^{[\mu} B^{n\ \nu\lambda]} - \sigma^2 B_{\mu\nu}^n B^{n\ \mu\nu} + C B_{\mu\nu}^n \tilde{B}^{n\ \mu\nu} \quad (29)$$

where $\tilde{B}^{n\ \mu\nu}$ ($= \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}^n$) is the dual of Kalb-Ramond field and σ and C are two constants defined by

$$\frac{e^{-2\sigma}}{r_c^2} \frac{1}{\chi^n} \partial_\phi \partial_\phi \chi^n = \sigma^2$$

and

$$\frac{e^{-2\sigma}}{r_c} \frac{M_0}{\chi^n} \partial_\phi \chi^n = C.$$

The equation of motion obtained from the KR action is

$$(\square - \sigma^2) B^{n\ \mu\nu} = C \tilde{B}^{n\ \mu\nu} \quad (30)$$

From now onwards we use the notation \star to denote the duality operation such that $\star B_{\mu\nu} = \tilde{B}_{\mu\nu}$. Then the above equation can be written as,

$$(\square - \sigma^2) B^{n\ \mu\nu} = C \star B^{n\ \mu\nu} \quad (31)$$

Next we make use of a well known result of differential geometry. Namely, on any manifold

$$\square \star = \star \square \quad (32)$$

That is to say, the duality operation commutes with the d'Alambertian. Now, applying $(\square - \sigma^2)$ once more on equation (31) and using the above result, we have,

$$\begin{aligned} (\square - \sigma^2)^2 B^{n\ \mu\nu} &= C(\square - \sigma^2) \star B^{n\ \mu\nu} \\ &= C \star (\square - \sigma^2) B^{n\ \mu\nu} \\ &= C^2 B^{n\ \mu\nu} \end{aligned} \quad (33)$$

which means,

$$(\square - \sigma^2 + C)(\square - \sigma^2 - C)B^{n\ \mu\nu} = 0 \quad (34)$$

Keeping in mind that the two factors on the left hand side of the above equation commutes, several possibility emerges out of the above equation. We take them up one by one.

Possibility I:

$$(\square - \sigma^2 - C)B^{n\ \mu\nu} = 0 \quad (35)$$

which implies

$$B^{n\ \mu\nu} = \star B^{n\ \mu\nu} \quad \text{i.e, } B^{n\ \mu\nu} \text{ is selfdual.} \quad (36)$$

Possibility II:

$$(\square - \sigma^2 + C)B^{n\ \mu\nu} = 0 \quad (37)$$

implies

$$-B^{n\ \mu\nu} = \star B^{n\ \mu\nu} \quad \text{i.e, } B^{n\ \mu\nu} \text{ is anti selfdual.} \quad (38)$$

Possibility III: The third possibility arises when none of the factors in equation (34) operating on $B^{n\ \mu\nu}$ gives zero, yet the product is zero. This could happen with operators. In such case both of the following conclusion will hold.

(a) Defining

$$(\square - \sigma^2 + C)B^{n\ \mu\nu} = F \quad (39)$$

one gets

$$\begin{aligned} C \star B^{n\ \mu\nu} + C B^{n\ \mu\nu} &= F \\ F &= \star F \end{aligned} \quad (40)$$

Now we take the dual of equation (39)

$$(\square - \sigma^2) \star B^{n\ \mu\nu} + C \star B^{n\ \mu\nu} = \star F \quad (41)$$

But equation (40) and (41) together mean

$$\begin{aligned} (\square - \sigma^2 + C)B^{n\ \mu\nu} &= (\square - \sigma^2 + C) \star B^{n\ \mu\nu} \\ \text{i.e, } B^{n\ \mu\nu} &= \star B^{n\ \mu\nu} \end{aligned} \quad (42)$$

(b) At the same time we define

$$(\square - \sigma^2 - C)B^{n\ \mu\nu} = F' \quad (43)$$

Similarly one gets

$$\begin{aligned} C \star B^{n\ \mu\nu} - C B^{n\ \mu\nu} &= F' \\ \star F' &= - F' \end{aligned} \quad (44)$$

Again taking the dual of equation (43)

$$(\square - \sigma^2) \star B^{n\ \mu\nu} - C \star B^{n\ \mu\nu} = \star F' \quad (45)$$

which together with (44) implies

$$\begin{aligned} -(\square - \sigma^2 - C)B^{n\ \mu\nu} &= (\square - \sigma^2 - C) \star B^{n\ \mu\nu} \\ \text{i.e,} \quad -B^{n\ \mu\nu} &= \star B^{n\ \mu\nu} \end{aligned} \quad (46)$$

Now from (a) and (b) above, if both the equation (42) and (46) has to follow the only conclusion is that $B^{n\ \mu\nu} = 0$. Note that this will not be the case if either F or F' is zero, in which case we get back the possibility I or II. So, combining all the possibilities, we could conclude that with the term $C B_{\mu\nu} \tilde{B}^{n\ \mu\nu}$ present in the 4 dimensional lagrangian we must have,

$$\begin{aligned} \tilde{B}^{n\ \mu\nu} &= B^{n\ \mu\nu} \\ \tilde{B}^{n\ \mu\nu} &= - B^{n\ \mu\nu} \\ B^{n\ \mu\nu} &= 0 \end{aligned} \quad (47)$$

That is to say the only nontrivial solutions correspond to selfdual or anti-selfdual Kalb-Ramond fields.

References

- [1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998); Phys.Rev. D.59, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436, 257 (1998).
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); *ibid*, 83, 4690 (1999).
- [3] W.D.Goldberger and M.B.Wise, Phys.Rev.D60, 107505 (1999).
- [4] H.Davoudiasl, J.L.Hewett and T.G.Rizzo, Phys. Lett. B473, 43 (2000).
- [5] Y.Grossman and M.Neubert, Phys. Lett. B474, 361 (2000).
- [6] K.Agashe, N.G.Deshpande and G.H.Wu, Phys. Lett. B511, 85 (2001).
- [7] N.Arkani-Hamed, S. Dimopoulos, G. Dvali and J.M.Russell, Phys. Rev. D65, 024032 (2002).

- [8] L. N. Chang, O. Lebedev, W. Loinaz and T. Takeuchi, Phys. Rev. Lett. 85, 3765 (2000).
- [9] O. Lebedev, *hep-ph/0201125*.
- [10] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys. Rev. D65, 124021 (2002).
- [11] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys. Rev. Lett. 89, 121101 (2002).
- [12] *Superstring Theory*, M. B. Green, J. H. Schwarz and E. Witten, Cambridge University Press, Cambridge (1987);
- [13] F. Hehl *et al.*, Rev. of Mod. Phys. 48, 393(1976); Phys. Rep, 258, 1 (1995); *Spin and Torsion in Gravitation*, V. De Sabbata and C. Sivaram, World Scientific, Singapore (1994).
- [14] M. Kalb and P. Ramond, Phys. Rev. D9, 2273 (1974).
- [15] P. Majumdar and S. SenGupta, Class. Quant. Grav. 16, L89 (1999). *String Theory*, J. Polchinski, Cambridge University Press., Cambridge, (1998).
- [16] R. Balbinot, J. C. Fabris and R. Kerner, Phys. Rev. D42, 1023, (1990); C. M. Chen, T. Harko and M. K. Mak, Phys. Rev. D61, 104017 (2000).
- [17] S. SenGupta and A. Sinha, Phys. Lett. B 514, 109 (2001); A. Lue, L. Wang and M. Kamionkowski, Phys. Rev. Lett. 83, 1506 (1999); N. Lepora, gr-qc/9812077 ; D. Maity, P. Majumdar, S. SenGupta, hep-th/0401218.
- [18] *Gravitation and Cosmology*, S. Weinberg, John Wiley & Sons, U.S.A (1972);
- [19] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. 84, 2080 (2000).
- [20] B. Mukhopadhyaya and S. SenGupta, Phys. Lett. B458, 8 (1999).
- [21] B. Mukhopadhyaya, S. Sen, S. Sur and S. SenGupta, *hep-th/0207165*.